Fault Tolerant Homopolar Magnetic Bearings with Flux Invariant Control

Uhn Joo Na*

Division of Mechanical and Automation Engineering, Kyungnam University, Masan, Kyungnam 631-701, Korea

The theory for a novel fault-tolerant 4-active-pole homopolar magnetic bearing is developed. If any one coil of the four coils in the bearing actuator fail, the remaining three coil currents change via an optimal distribution matrix such that the same opposing pole, C-core type, control fluxes as those of the un-failed bearing are produced. The hompolar magnetic bearing thus provides unaltered magnetic forces without any loss of the bearing load capacity even if any one coil suddenly fails. Numerical examples are provided to illustrate the novel fault-tolerant, 4-active pole homopolar magnetic bearings.

Key Words : Magnetic Bearings, Fault Tolerance, Active Vibration Control, Rotor Dynamics, Mechatronics, Magnetic Suspension

Nomenclature

μ_0 : Permeability of air
a_0 : Pole face area of an active pole
g_0 : Nominal air gap in active pole plane
x, y : Journal displacements.
H_c : Coercive force of a permanent magnet
L_{pm} : Length of a permanent magnet
ς : Flux fringing factor
ϕ_j : Flux through the j-th pole
i_j : Currents through the j-th pole
n : Number of coil turns
R_j : Air gap reluctance
R_R : Permanent magnet reluctance
v_{cx}, v_{cy} : x and y control voltages
Ω : Rotating speed
θ : Pole angle
I : Current vector
T : Current distribution matrix
K_p, K_d : Proportional and derivative gains
λ : Lagrange Multiplier

* **E-mail** : uhnjoona@kyungnam.ac.kr
TEL : +82-55-249-2162; **FAX** : +82-55-249-2617
Division of Mechanical and Automation Engineering, Kyungnam University, Masan, Kyungnam 631-701, Korea. (Manuscript **Received** October 12, 2005; **Revised** February 22, 2006)

1. Introduction

A magnetic bearing system is an active control device that suspends the spinning rotor magnetically without physical contact as well as suppresses vibrations. Magnetic bearings find greater use in high speed, high performance applications such as flywheel energy storage systems since they have many advantages over rolling element bearings (Bitterly, 1997). Unlike electromagnetic heteropolar bearings shown in the previous works (Allaire et al., 1983 ; Salm and Schweitzer, 1984 ; Matsumura and Yoshimoto, 1986 ; Ahn and Han, 2003 ; Na, 2005), permanent magnet biased homopolar magnetic bearings have a unique biasing scheme that directs the bias flux flow into the active pole plane where it energizes the working air gaps, and then returns through the dead pole plane and the shaft sleeve. Use of rare earth permanent magnets such as Samarium-Cobalt and Neodymium-Iron-Boron yields a very high efficiency when the permanent magnets are used as the source of bias flux to energize the air gaps and electromagnets are used to supply control fluxes in the active plane. Sortore et al. (1990) and Allaire et al. (1992) presented design methods and experimental verifications for permanent magnet biased magnetic bearings. Similarly, Maslen et al. (1996) also showed both analytical and experimental results on the design and construction of permanent magnet biased magnetic bearings. Lee et al. (1994) provides design, testing and performance limits of the permanent magnet biased magnetic bearings. Fan et al. (1997) presented systematic design procedures for permanent magnet biased magnetic bearings. Fukata and Yutani (1998) studied the frequency response of permanent magnet biased magnetic bearings.

Fault-tolerance of the magnetic bearing system is of great concern for highly critical applications of turbo-machinery since a failure of any one control components may lead to the complete system failure. Maslen and Meeker (1995) introduced a fault tolerance of an 8-pole heteropolar magnetic bearing actuator with independently controlled currents and experimentally verified it (Maslen et al., 1999). They utilized flux coupling in a heteropolar magnetic bearing that allows the remaining coils to produce force resultants identical to the unfailed bearing. Na and Palazzolo (2001) provided the optimized realization of fault-tolerant magnetic bearing actuators so that fault-tolerant control can be realized for an 8-pole bearing for up to 5 coils failed, and experimentally verified it (Na et al., 2002).

Fault tolerant scheme applied to heteropolar magnetic bearings was modified and extended to permanent magnet biased homopolar magnetic bearings in Na's work (Na, 2004). He presented the optimized realization of fault tolerant 8-pole homopolar magnetic bearings, so that the same magnetic forces can be realized up to any combination of 5 coils failed out of 8 coils. However, the overall load capacity of the bearing is reduced as coils fail. The same magnetic forces are then preserved up to the load capacity of the failed bearing.

The present work extends Na's previous result (Na, 2004) such that unique fault tolerant schemes are realized for 4 active pole homopolar magnetic bearings. The theory and numerical analysis for the novel fault-tolerant homopolar magnetic bearings are presented. The bearing can preserve the same C-core fluxes and magnetic forces identical to the unfailed bearing even after any one coil out of four coils fails. The overall load capacity of the bearing actuator thus remains the same as before and after failure.

2. Bearing Model

The schematic drawing of a 4-active pole, permanent magnet biased homopolar magnetic bearing is shown in Fig. 1. Assuming that eddy current effects and material path reluctances are neglected, Maxwell's equations are reduced to the equivalent magnetic circuit for the homopolar magnetic bearing as shown in Fig. 2. The reluctance in air gap j of the active pole plane is;

$$R_j = \frac{g_j}{\mu_0 a_0} \tag{1}$$

where

$$g_j = g_0 - x\cos\theta_j - y\sin\theta_j \tag{2}$$

The permanent magnets, which are modeled as a source $H_c L_{pm}$ and the return path reluctance R_R, provide bias flux to the working air gaps. Applying Ampere's loop law to the magnetic circuit results in 4 independent equations.

$$R_j\phi_j - R_{j+1}\phi_{j+1} = ni_j - ni_{j+1},\ j=1,2,3 \tag{3}$$

$$R_j\phi_j + R_R\phi_b = H_c L_{pm} + ni_j,\ j=4 \tag{4}$$

Applying flux conservation law to the magnetic circuit results in one more equation.

$$\phi_1 + \phi_2 + \phi_3 + \phi_4 = \phi_b \tag{5}$$

Rearranging Eqs. (3)-(5) leads to a matrix equation.

$$\begin{bmatrix} R_1 & -R_2 & 0 & 0 \\ 0 & R_2 & -R_3 & 0 \\ 0 & 0 & R_3 & -R_4 \\ 1 & 1 & 1 & 1+\frac{R_4}{R_R} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{H_c L_{pm}}{R_R} \end{bmatrix} + \begin{bmatrix} n & -n & 0 & 0 \\ 0 & n & -n & 0 \\ 0 & 0 & n & -n \\ 0 & 0 & 0 & \frac{n}{R_R} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \tag{6}$$

or

$$R\Phi = H + NI \tag{7}$$

The 4-active pole homopolar magnetic bearing utilizes 4 independent coils each driven by its power amplifiers so as to obtain the flexibility of a coil failure. Control fluxes through poles are then fully coupled with independent currents. The

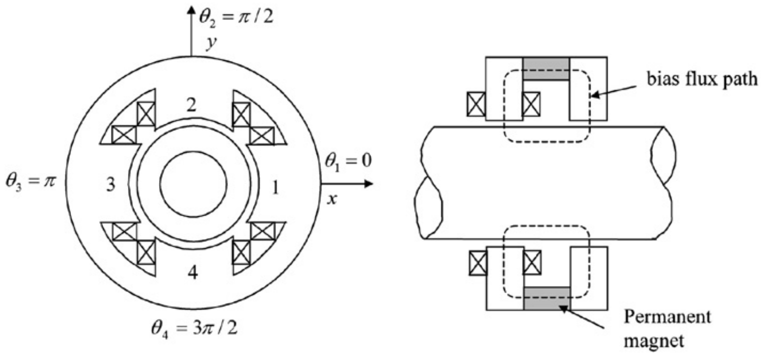

Fig. 1 Permanent magnet biased homopolar magnetic bearing

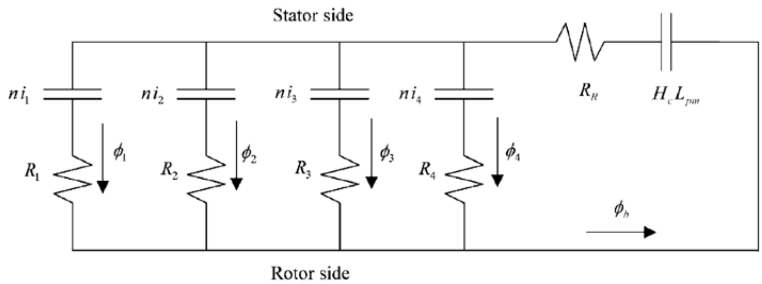

Fig. 2 Equivalent magnetic circuit for a permanent magnet biased homopolar bearing

currents distributed to the 4-active-pole bearing are generally expressed as a 4×2 distribution matrix T and control voltage vector v_c. The current vector is;

$$I = Tv_c \tag{8}$$

where

$$T = \lfloor T_x \ T_y \rfloor,\ v_c = \begin{bmatrix} v_{cx} \\ v_{cy} \end{bmatrix}$$

A typical current distribution scheme (coil winding scheme) for a homopolar magnetic bearing shown in Fig. 1 is;

$$\widetilde{T} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{9}$$

The feedback control voltages v_{cx} and v_{cy}, determined with any type of control law and measured rotor motions, are distributed to each pole via $\widetilde{T}$ in normal operation, and create effective stiffness and damping of the bearing to suspend the rotor around the bearing center position. If one of the 4 coils fails, the full (4×1) current vector is related to the reduced current vector by introducing a failure map matrix W.

$$I = W\hat{I} \tag{10}$$

For example the matrix W for the 4$^{\text{th}}$ coil failed bearing is described as;

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The reduced current vector $\hat{I}$ is defined as;

$$\hat{I} = \hat{T} v_c \tag{11}$$

where

$$\begin{gathered} \hat{T} = \lfloor \hat{T}_x \ \hat{T}_y \rfloor \\ \hat{T}_x = [t_1,\ t_2,\ t_3]^T,\ \hat{T}_y = [t_4,\ t_5,\ t_6]^T \end{gathered} \tag{12}$$

The goal of the present work is to determine the optimal reduced distribution matrix $\hat{T}$ such that the magnetic forces should remain very much invariant before and after failure.

The flux density vector can be determined from the magnetic circuit equation in Eq. (7). Leakage and fringing effects can be empirically determined and are simply derated by the fringing factor ς (Allaire et al., 1989). The flux density vector in the air gaps is described as;

$$B=\varsigma A^{-1}R^{-1}(H+NI) \tag{13}$$

where the pole face area matrix is $A=diag([a_0, a_0, a_0, a_0])$. The flux density vector is then reformulated as;

$$B=Gv \tag{14}$$

where

$$G=\lfloor G_bH\ G_c\hat{T}_x\ G_c\hat{T}_y \rfloor,\ v=\begin{bmatrix} 1 \\ v_{cx} \\ v_{cy} \end{bmatrix}$$
$$G_b=\varsigma A^{-1}R^{-1} \tag{15}$$
$$G_c=\varsigma A^{-1}R^{-1}NW$$

The magnetic forces developed in the active pole plane are described as;

$$f_x=B^T\frac{\partial D}{\partial x}B \tag{16}$$

$$f_y=B^T\frac{\partial D}{\partial y}B \tag{17}$$

where the air gap energy matrix is;

$$D=diag([g_ja_0/(2\mu_0)]) \tag{18}$$

The magnetic forces are then expressed as;

$$f_x=v^TM_xv \tag{19}$$

$$f_y=v^TM_yv \tag{20}$$

where

$$M_x(\hat{T})=-G^T\frac{\partial D}{\partial x}G$$
$$M_y(\hat{T})=-G^T\frac{\partial D}{\partial y}G \tag{21}$$

3. Optimization of Fault Tolerant Magnetic Forces

3.1 Bias linearization of magnetic forces

The magnetic forces in Eqs. (19) and (20) can be linearized about the bearing center position even though any one coil fail. The linearized magnetic forces are;

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix}=-\begin{bmatrix} k_{pxx} & k_{pxy} \\ k_{pyx} & k_{pyy} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} k_{vxx} & k_{vxy} \\ k_{vyx} & k_{vyy} \end{bmatrix}\begin{bmatrix} v_{cx} \\ v_{cy} \end{bmatrix} \tag{22}$$

where the position stiffnesses are defined as;

$$k_{p\varphi\omega}=-H^T\frac{\partial Q_{\varphi b}}{\partial\omega}\bigg|_{\substack{\varphi=0\\ \omega=0}}H$$
$$Q_{\varphi b}=-G_b\frac{\partial D}{\partial\varphi}G_b \tag{23}$$

and where the voltage stiffnesses are defined as;

$$k_{v\varphi\omega}=2H^TQ_{b\varphi}|_{\substack{\varphi=0\\ \omega=0}}\hat{T}_\omega$$
$$Q_{b\varphi}=-G_b\frac{\partial D}{\partial\varphi}G_c \tag{24}$$

The parameters φ and ω represent either x or y. The position stiffnesses of the homopolar bearing remain unchanged with a coil failure since the position stiffnesses are only influenced by the bias flux driven with permanent magnets.

Employing an optimal current distribution matrix T may help to maintain the same decoupled magnetic forces as those of an unfailed magnetic bearing (Maslen et al., 1995; Na and Palazzolo, 2001). The necessary conditions to yield the same decoupled magnetic control forces are given as (Na, 2004);

$$M_x=\begin{bmatrix} 0 & k_v/2 & 0 \\ k_v/2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ M_y=\begin{bmatrix} 0 & 0 & k_v/2 \\ 0 & 0 & 0 \\ k_v/2 & 0 & 0 \end{bmatrix} \tag{25}$$

where the direct voltage stiffness for an unfailed bearing is defined as;

$$k_v=k_{v\varphi\varphi}=2H^TQ_{b\varphi}|_{\varphi=0}T_\varphi \tag{26}$$

If the distribution matrix $\hat{T}$ is determined such that Eq. (21) satisfies Eq. (25), the nonlinear magnetic forces in Eqs. (19) and (20) become linearized at bearing center positions such as;

$$f_x=k_vv_{cx},\ f_y=k_vv_{cy} \tag{27}$$

Equations (21) and (25) can be written in 18 scalar forms, and then boils down to 10 algebraic equations if redundant terms are eliminated. The necessary conditions to yield the same control forces before and after failure are then described as;

$$
\begin{aligned}
&h_1(\hat{T}) = \hat{T}_x^T Q_{x0} \hat{T}_x = 0 \\
&h_2(\hat{T}) = \hat{T}_x^T Q_{x0} \hat{T}_y = 0 \\
&h_3(\hat{T}) = H^T Q_{bx0} \hat{T}_y = 0 \\
&h_4(\hat{T}) = \hat{T}_x^T Q_{x0} \hat{T}_y = 0 \\
&h_5(\hat{T}) = H^T Q_{bx0} \hat{T}_x = k_v/2 \\
&h_6(\hat{T}) = \hat{T}_x^T Q_{y0} \hat{T}_x = 0 \\
&h_7(\hat{T}) = \hat{T}_y^T Q_{y0} \hat{T}_y = 0 \\
&h_8(\hat{T}) = H^T Q_{by0} \hat{T}_x = 0 \\
&h_9(\hat{T}) = \hat{T}_x^T Q_{y0} \hat{T}_y = 0 \\
&h_{10}(\hat{T}) = H^T Q_{by0} \hat{T}_y = k_v/2
\end{aligned}
\tag{28}
$$

where

$$Q_{\varphi b0} = -G_b \frac{\partial D}{\partial \varphi} G_b \bigg|_{\substack{\varphi=0 \\ \omega=0}}, \quad Q_{\varphi 0} = -G_c \frac{\partial D}{\partial \varphi} G_c \bigg|_{\substack{\varphi=0 \\ \omega=0}}$$

A distribution matrix $\hat{T}$ can be determined by using the Lagrange Multiplier method to minimize the Euclidean norm of the flux density vector B (Na and Palazzolo, 2001). The cost function is defined as;

$$J = B(\hat{T})^T P B(\hat{T}) \tag{29}$$

where the diagonal weighting matrix P is also selected to maximize the load capacity. The Lagrange Multiplier method is then used to solve for $\hat{T}$ that satisfies Eq. (29). Define:

$$L(\hat{T}) = B(\hat{T})^T P B(\hat{T}) + \sum_{j=1}^{10} \lambda_j h_j(\hat{T}) \tag{30}$$

Partial differentiation of Eq. (30) with respect to t_i and λ_j leads to 16 nonlinear algebraic equations to solve for t_i and λ_j.

$$\Psi = \begin{bmatrix} \psi_1(t, \lambda) \\ \psi_2(t, \lambda) \\ \vdots \\ \psi_{15}(t, \lambda) \\ \psi_{16}(t, \lambda) \end{bmatrix} = 0 \tag{31}$$

where

$$\psi_i = \frac{\partial L}{\partial t_i} = 0, \; i = 1, 2, \cdots, 6 \tag{32}$$

$$\psi_{2q+j} = h_j(\hat{T}) = 0, \; j = 1, 2, \cdots, 10 \tag{33}$$

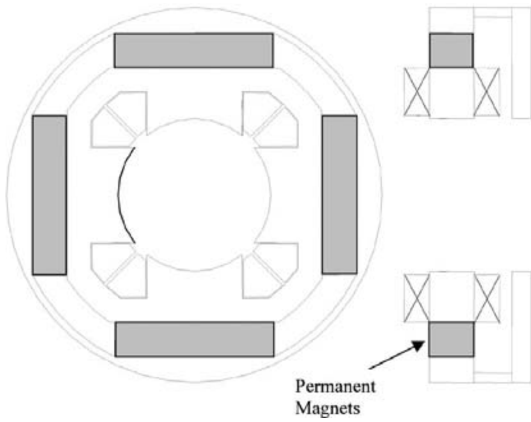

Fig. 3 Designed homopolar magnetic bearing

3.2 Optimal realization of distribution matrices

The following examples illustrate optimal realization of distribution matrix T for a 4-active pole homopolar magnetic bearing. The designed 4-pole permanent magnet biased magnetic bearing has the bearing parameters a_0 (0.0004537 m^2), g_0 (0.000508 m), and n (70 turns). The bias flux density in the air gaps at rotor center position is assumed to be 0.6 tesla. The voltage stiffness k_v is then calculated as 48.013 N/volt.

Optimal distribution matrices are determined for the 4 cases of coil failures of the magnetic bearing in Fig. 1. Various initial guesses of t_i and λ_j are tested to find the solution of Eq. (31). The distribution matrix solution for the 1st coil failed bearing is determined as;

$$T_1 = \begin{bmatrix} 0 & 0 \\ -1 & 1 \\ -2 & 0 \\ -1 & -1 \end{bmatrix} \tag{34}$$

Equations (21) with the solution T_1 at the rotor center positions are;

$$
\begin{aligned}
M_x &= \begin{bmatrix} 0 & 24.0065 & 0 \\ 24.0065 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
M_y &= \begin{bmatrix} 0 & 0 & 24.0065 \\ 0 & 0 & 0 \\ 24.0065 & 0 & 0 \end{bmatrix}
\end{aligned}
\tag{35}
$$

The optimal distribution matrix solution T_1 for the 1st coil failed operation satisfies the necessary conditions in Eq. (25) to yield the same magnetic control forces. This means that the same magnetic control forces as those before failure are guaranteed even if T_1 is replaced for $\widetilde{T}$. Similarly, the optimal distribution matrix solutions for the 2nd, 3rd, and the 4th coil failed operations are calculated by using Eq. (31) such as ;

$$T_2=\begin{bmatrix} 1 & -1 \\ 0 & 0 \\ -1 & -1 \\ 0 & 2 \end{bmatrix},\ T_3=\begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & -1 \end{bmatrix},\ T_4=\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ -1 & 1 \\ 0 & 0 \end{bmatrix} \quad (36)$$

where T_i corresponds to pole i being failed in Fig. 1.

It is interesting to note that the calculated voltage stiffnesses of k_{vxx}, k_{vxy}, k_{vx}, and k_{vyy} for $\widetilde{T}$, T_1, T_2, T_3, and T_4 are all identical to 48.013 N/volt, 0 N/volt, 0 N/volt, and 48.013 N/volt, respectively. This means that the same linearized dynamic properties can be maintained before and after any one coil fails for the homopolar bearing if the distribution matrix T_1, T_2, T_3, and T_4 is appropriately replaced for $\widetilde{T}$.

4. Numerical Analysis

With the uniform current distribution with $\widetilde{T}$ as well as the symmetric bearing geometries, magnetic forces are decoupled and vary linearly with respect to control currents and rotor displacements around the bearing center position. If symmetry is lost due to a coil failure, magnetic forces are no longer decoupled and linear with respect to control currents and rotor displacements, and even it may be difficult to maintain stable control. Reassigning the remaining currents with a redifined current distribution scheme may remedy this by providing the same decoupled magnetic forces as those before failure. Numerical analysis is given in this section to verify the new fault tolerant theory.

4.1 Three-dimensional magnetic field analysis

Magnetic fluxes are analyzed for the failed magnetic bearing. The 3-D magnetic field model is constructed for a homopolar magnetic bearing shown in Fig. 7. The designed 4-pole permanent magnet biased magnetic bearing has the bearing

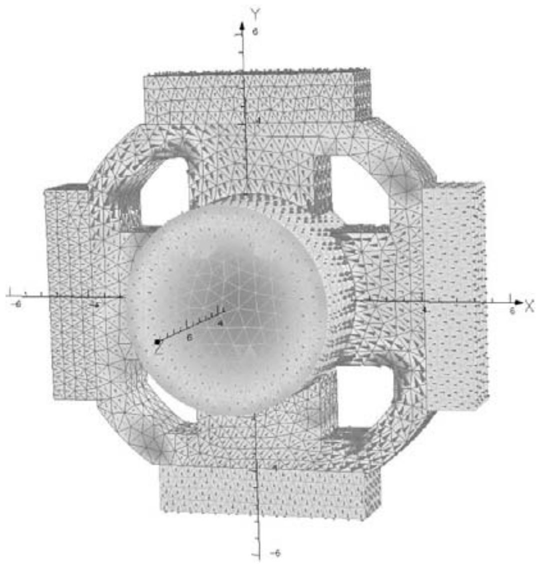

Fig. 4 Flux distribution calculated with 3-D finite element model by current set $I_2|_{t=\pi/(6\Omega)}$

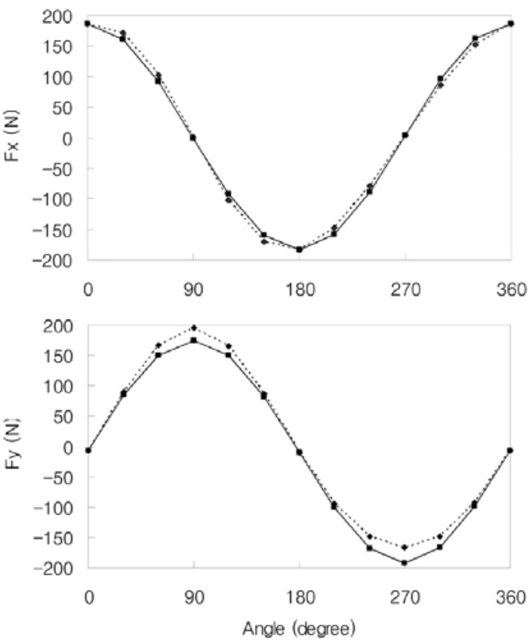

Fig. 5 Magnetic Forces Calculated with 3-D Finite Element Model by I_1 (solid lines), and I_2 (dashed lines)

parameters a_0(0.0004537 m^2), g_0(0.000508 m), and n(70 turns). A commercial magnetic field software (OPERA3D) is used for the 3-D field calculation.

12 current sets of $I_1=\widetilde{T}v_c$ and $I_2=T_4v_c$, $v_{cx}=3.0\cos\Omega t$, $v_{cy}=30\sin\Omega t$, at $t=[\pi/(6\Omega), 2\pi/(6\Omega), \cdots, 12\pi/(6\Omega)]$ are applied on the model. The corresponding magnetic fluxes are then calculated at each current sets. Flux distribution in the active pole plane driven with the current set $I_1|_{t=\pi/(6\Omega)}$ is shown in Fig. 4. The flux distribution driven with $I_2|_{t=\pi/(6\Omega)}$ is also calculated and turned out to be almost identical to that with $I_1|_{t=\pi/(6\Omega)}$. The magnetic forces are also calculated with 12 current sets of I_1 and I_2. Figure 5 shows that the magnetic forces driven by I_1 are very much the same as those driven by I_2. However, there exist slight differences between the magnetic forces driven by I_1 and I_2. The control fluxes in the active pole plane are not completely isolated from the return pole plane. Thus some amount of control fluxes leak through the return pole plane.

It is concluded from the 3-D simulation that the unique current distribution $I_2=T_4v_c$ for the 4^{th} coil failed bearing drives very much the same C-core control fluxes as well as magnetic forces as those of the current $I_1=\widetilde{T}v_c$ for the unfailed bearing.

4.2 Dynamic simulations

The fault tolerant control system consists of two independent parts, which are a feedback voltage control law and an adaptive current distribution mechanism. A simple PD feedback control law is used to stabilize the system. While the feedback control law remains unaltered during the failure the appropriate current distribution matrix T can be continuously updated according to the failure status of coils (Na et al., 2002). Schematic of the fault tolerant control system is shown in Fig. 6.

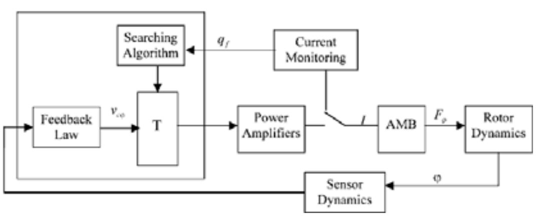

Fig. 6 Schematic of the fault-tolerant controller

The fault-tolerant control system is simulated on a horizontal rigid rotor supported on the 4-active pole homopolar bearings. The sy mmetric rotor used in this analysis has mass of 12 kg, the polar moment of inertia of 0.05 kgm^2, and the transverse moment of inertia of 0.36 kgm^2 about the mass center. Two magnetic bearings are located at 0.22 m either side of the mass center. An unbalance eccentricity of 1.0×10^{-5} m is applied at both bearing locations with relative phase angle 90°. The power amplifiers are simply modeled with the DC gain of 1 Ampere/Volt. The sensors are also simply modeled with a DC sensor gain of 7874 volt/m. The closed loop bearng stiffness and damping can be adjusted by tuning the PD control gains (Keith et al., 1990). The designed PD controller has the proportional gain of 60 and the derivative gain of 0.05. The proportional control gain of K_p is tuned to produce the positive closed loop bearing stiffness. The derivative control gain of K_d is tuned to add enough closed loop bearing damping.

The transient response from normal operation with no failure to fault-tolerant control with the 4^{th} coil failed for both bearings was simulated for nonlinear force modeled bearings at 10,000 RPM. The nonlinear magnetic bearings are used for a

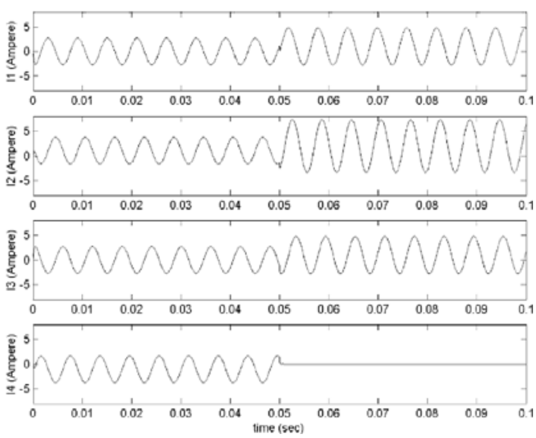

Fig. 7 Current plot for normal operation to the 4^{th} coil failed operation

more stringent test. Figure 7 shows transient current inputs from the normal unfailed operation through failure of the 4^{th} coil of the bearing A failed at 0.05 sec. Transient response of the corresponding flux densities to the bearing A is shown in Fig. 8. The transient response of the displacements at the bearing A is shown in Fig. 9. Displacements and flux density variation at Bearing A remain unchanged even after the 4^{th} coil fails, while the three remaining currents are redistributed via T_4 after the 4^{th} coil fails at 0.05 seconds. This means that any one coil out of four coils is free to fail while bearing properties such as the load capacity and stiffness remain invariant, if $\widetilde{T}$ is replaced by T_1, T_2, T_3, and T_4 shortly after failure.

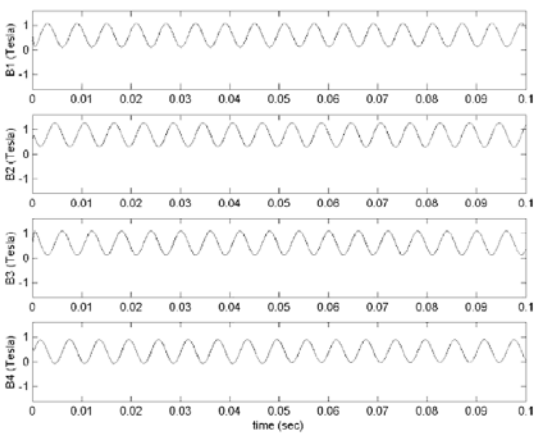

Fig. 8 Flux density plot for normal operation to the 4^{th} coil failed operation

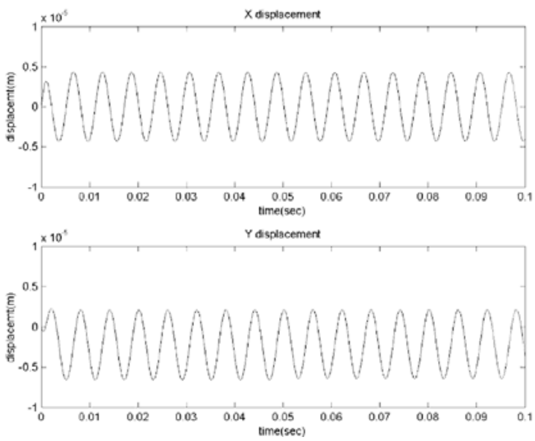

Fig. 9 Displacement plot for normal operation to the 4^{th} coil failed operation

5. Conclusions

A unique fault-tolerant control scheme is developed for the permanent magnet biased, 4-active-pole homopolar magnetic bearings. The fault tolerant control scheme is that, if any one coil among four coils in the bearing suddenly fails, the remaining three coil currents are redistributed by a distribution matrix such that the same control fluxes as those of an un-failed bearing are preserved. The necessary conditions for the fault tolerant magnetic bearing are developed. The Lagrange Multiplier optimization method with equality constraints is utilized to calculate the optimal distribution matrices.

The fault tolerant control scheme is verified numerically on the three dimensional magnetic bearing flux model. The magnetic forces calculated with the 3-D finite element model are well matched with the theory. The fault-tolerant control system is also simulated on a horizontal rigid rotor supported on the 4-active pole homopolar bearings. Dynamic simulation shows that very much the same vibrations as well as the same fluxes are maintained throughout the failure event.

The advantages of the new fault-tolerant control scheme over the previous results (Na, 2004) are that the magnetic bearing can preserve the same C-core fluxes identical to the unfailed bearing even after any one coil among four coils fails. Thus the magnetic bearing utilizing the fault tolerant scheme has the redundancy of a coil failure without sacrificing the load capacity, position stiffness or current stiffness. The energy efficient homopolar magnetic bearings with fault tolerant capability may find great use in some applications of the high speed, high performance rotating machinery.

Acknowledgments

This work was supported by Kyungnam University research fund.

References

Ahn, H. and Han, D., 2003, "System Modeling and Robust Control of an AMB Spindle Part II A Robust Controller Design and its Implementation," *KSME International Journal*, Vol. 17, No. 12, pp. 1855～1866.

Allaire, P. E., Lewis, D. W. and Knight, J. D., 1983, "Active Vibration Control of a Single Mass Rotor on Flexible Supports," *Journal of the Franklin Institute*, Vol. 315, pp. 211～222.

Allaire, P. E., Maslen, E. H., Humphris, R. R., Sortore, C. K. and Studer, P. A., 1992, "Low Power Magnetic Bearing Design for High Speed Rotating Machinery," *Proceedings of the NASA International Symposium on Magnetic Suspension Technology*, pp. 317～329.

Allaire, P. E., Mikula, A., Banerjee, B. B., Lewis, D. W. and Imlach, J., 1989, "Design and Test of a Magnetic Thrust Bearing," *Journal of the Franklin Institute*, Vol. 326, pp. 831～847.

Bitterly, J. G, and Bitterly, S. E., 1997, "Flywheel Based Energy Storage System," U.S. Patent No. 5,614,777.

Fan, Y., Lee, A. and Hsiao, F., 1997, "Design of a Permanent/Electromagnetic Magnetic Bearing-Controlled Rotor System," *Journal of the Franklin Institute*, Vol. 334B, pp. 337～356.

Fukata, S. and Yutani, K., 1998, "Characteristics of Electromagnetic Systems of Magnetic Bearings Biased with Permanent Magnets," *Proceedings of the Sixth International Symposium on Magnetic Bearings*, pp. 234～243.

Keith, F. J., Williams, R. D. and Allaire, P. E., 1990, "Digital Control of Magnetic Bearings Supporting a Multimass Flexible Rotor," *STLE Tribology Transactions*, Vol. 33, pp. 307～314.

Lee, A. C., Hsiao, F. Z. and Ko, D., 1994, "Analysis and Testing of a Magnetic Bearing with Permanent Magnets for Bias," *JSME International Journal, Series C*, Vol. 37, pp. 774～782.

Lee, A. C., Hsiao, F. Z. and Ko, D., 1994, "Performance Limits of Permanent-Magnet-Biased Magnetic Bearings," *JSME International Journal, Series C*, Vol. 37, pp. 783～794.

Maslen, E. H., Allaire, P. E., Noh, M. D. and Sortore, C. K., 1996, "Magnetic Bearing Design for Reduced Power Consumption," *ASME Journal of Tribology*, Vol. 118, pp. 839～846.

Maslen, E. H., Meeker, D. C., 1995, "Fault Tolerance of Magnetic Bearings by Generalized Bias Current Linearization," *IEEE Transactions on Magnetics*, Vol. 31, pp. 2304～2314.

Maslen, E. H., Sortore, C. K., Gillies, G. T., Williams, R. D., Fedigan, S. J. and Aimone, R. J., 1999, "A Fault Tolerant Magnetic Bearings," *ASME Journal of Engineering for Gas Turbines and Power*, Vol. 121, pp. 504～508.

Matsumura, F. and Yoshimoto, T., 1986, "System Modeling and Control of a Horizontal-Shaft Magnetic-Bearing System," *IEEE Transactions on Magnetics*, Vol. 22, pp. 197～206.

Na, U. J., 2004, "Fault Tolerance of Homopolar Magnetic Bearings," *Journal of Sound and Vibration*, Vol. 272, pp. 495～511.

Na, U. J., 2005, "Fault Tolerant Control of Magnetic Bearings with Force Invariance," *Journal of Mechanical Science and Technology*, Vol. 19. pp. 731～742.

Na, U. J. and Palazzolo, A. B., 2001, "Fault Tolerance of Magnetic Bearings with Material Path Reluctances and Fringing Factors," *IEEE Transactions on Magnetics*, Vol. 36, pp. 3939～3946.

Na, U. J., Palazzolo, A. B. and Provenza, A., 2002, "Test and Theory Correlation Study for a Flexible Rotor on Fault-Tolerant Magnetic Bearings," *ASME Journal of Vibration and Acoustics*, Vol. 124, pp. 359～366.

Salm, J. and Schweitzer, G., 1984, "Modeling and Control of a Flexible Rotor with Magnetic Bearing," *Proceedings of the Third International Conference on Vibrations in Rotating Machinery*, pp. 553～561.

Sortore, C. K., Allaire, P. E., Maslen, E. H., Humphris, R. R. and Studer, P. A., 1990, "Permanent Magnet Biased Magnetic Bearings — Design, Construction and Testing," *Proceedings of the Second International Symposium on Magnetic Bearings*, pp. 175～182.